3. S. Putterman, Hydrodynamics of Superfluid Fluids [Russian translation], Mir, Moscow (1964).
4. S. F. Edwards and J. B. Taylor, "Negative temperature states of two-dimensional plasmas and vortex fluids," Proc. Roy. Soc. London, A336, 257 (1974).
5. N. V. Butenin, Yu. I. Neimark, and N. A. Fufaev, Introduction to the Theory of Nonlinear Oscillations [in Russian], Nauka, Moscow (1976).
6. H. Aref and N. Pomphrey, "Integrable and chaotic motions of four vortices," Proc. Roy. Soc. London, A380, 359 (1982).
7. E. A. Novikov, "Dynamics and statistics of a vortex system," Zh. Eksp. Teor. Fiz., 68, No. 5 (1975).
8. E. A. Novikov and Yu. B. Sedov, "Stochastic properties of a system of four vortices," Zh. Eksp. Teor. Fiz., 75, No. 3 (1978).
9. E. A. Novikov and Yu. B. Sedov, "Stochastic vortices," Pis'ma, Zh. Eksp. Teor. Fiz., 29, No. 12 (1979).
10. P. I. Geshev and A. I. Chernykh, Vortex Motion in a Two-Dimensional Simply Simply-Connected Domain. Preprint No. 65, Izd. Inst. Teor. Fiz. Sib. Otd. Akad. Nauk SSSR (1980).
11. H. Kirchhoff, Mechanics [Russian translation], Izd. Akad. Nauk SSSR, Moscow (1962).
12. E. J. Routh, "Some applications of conjugate functions," Proc. London Math. Soc., 12, 73 (1881).
13. C. C. Lin, On the Motion of Vortices in Two Dimensions, Univ. Toronto Press, Toronto (1943).
14. A. G. Sveshnikov and A. N. Tikhonov, Theory of Functions of a Complex Variable [in Russian], Nauka, Moscow (1967).
15. M. A. Lavrent'ev and B. V. Shabat, Methods of the Theory of Functions of a Complex Variable [in Russian], Nauka, Moscow (1965).

TWO TYPES OF VORTEX TUBE

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Experimental and theoretical modeling is an efficient method of solving problems of vortex generation [1-7].

It is noted that in a number of references that vertical flows in linear vortices are everywhere directed upwards, starting from the axis, right out to the boundary of the solid rotation, and only far away at the periphery does the direction change to the opposite [8, 9]. In other investiagtions the authors noted descending axial flows and flows ascending along the walls of the vertex tube [10, 11], and the vortex structure is called one-cell and two-cell, respectively. In [10] and attempt was made to classify vortex tubes, depending on the ratio of intensity of the vertical streams and the circulation. The present paper discusses an experimental study of transition from a one-cell to a two-cell vortex.

The vortex tubes are induced in a vortex chamber. The chamber height is 0.665 m. Its diameter is 0.382 m.

The upper part of the chamber has a four-blade vortex generator, mounted on the axis of a motor. The height of the rectangular blades was 0.07 m, and the width in some of the tests was 0.05 m, and in others 0.10 m. The generator was attached to the axis of the motor behind a heavy cylindrical platform. The platform diameter was 0.21 m, and its height was 0.05 m. The function of the platform was to deviate the flow, which arrived at the motor after interacting with the lower surface of the platform. In addition, this platform provided a stable frequency of rotation of the vortex generator. At the top the chamber was covered by a lid. Tests were conducted with the chamber both open and closed, and here an annular gap of the required dimensions was assigned by varying the diameter of the lid.

By using the gap in the lid we could change the dimensions of the zone in which the flow received angular momentum. In addition, as the gap changed there was a change in the

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flow rate Q per second along the axis of the vortex: $Q = 2\pi \int_0^R v_z r dr$, where R is the size of the ascending flow zone.

The motor frequency was monitored with a tachometer. It consisted of a light source, a photodiode, and an oscillograph. Measurements of velocity and pressure were made with a 5-channel spherical probe with sphere diameter 0.003 m, fastened to a rod of diameter 0.015 m.

The pressures in the probe apertures were measured with a Prandtl micromanometer. The probe was positioned at a fixed specific distance and could be moved along a radius, moved vertically, and rotated by an angle φ about the axis of the mounting rod.

In the flow the probe was set up at an angle φ such that the pressures in the aperatures 4 and 5, located transverse to the mounting bracket, became the same, i.e., $p_m = p_4 = p_5$. The measurements were recorded during the tests: the angle φ of the velocity vector relative to the chosen reference plane; the pressures p_1 and p_3 in the meridional apertures 1 and 3, and also the pressure p_2 in the central aperture 2.

The absolute values of velocity v, static pressure p_0, and flow downwash angle δ in the plane of the 1-2-3 meridian were later calculated from the equations: $q = kp_{2m}$, $p_0 = p_2 - f(\delta)q$, $\delta = cp_{31}/p_{2m}$, where $q = \rho v^2/2$; $p_{2m} = p_2 - p_m$; $p_{31} = p_3 - p_1$; $f(\delta)$, c, k are constants which were determined from the calibration curves.

The relative error of velocity measurement with the spherical probe, due to flow nonuniformity, $\delta_v = \frac{p_2 - p_m}{2p^*_{2m}} \frac{d}{y}$, where d is the distance between apertures 2 and 4 or 2 and 5 of the probe; p_{2m} is the pressure difference between apertures 2 and 4, which was noted in the region with considerable velocity gradients; y is the dimension of the zone with considerable drops in velocity (in our measurements y is the size of the vortex core).

According to our test data, even in situations least favorable for measurement (y = 0.017 m), the fall in pressure at the edge of the core, in comparison with the fall in pressure on the axis of the vortex, was $p^*_{2m} = 20$ scale divisions of the micromanometer. At distance d = 0.001 m the possible uncertainty in manometer readings due to flow nonuniformity was 1.2 divisions.

In the measurements at points far from the vortex axis, at distance (0.3-0.4)y , we have $p^*_{2m} \simeq 8$ divisions. As a result we find $\delta_v = (1{,}2/16)\,100\% = 7{,}5\%$. On the whole the errors of velocity measurement in cases unfavorable for measurement did not exceed 10%.

The vortex tubes generated in the chamber were distinguished by quite good stability. Visual observations showed only a slight deviation, in the range (0.2-0.3)D, of the end of the vortex from the axis. The frequency of these deviations was quite large and was not reflected in the micromanometer readings. The oscillation about the axis at the center of the vortex was considerably less than at the base. As a rule the measurements were conducted three times independently under the same conditions. The scatter of the measured data did not exceed 5% in the three measurements.

Amongst the large number of vortex tubes investigated two typical cases were observed. The qualitative picture of the flows in the vortex tubes was studied by visualizing the vaporization. Case 1 is a uniform distribution of condensate particles. Case 2 is a dense layer of particles forming the wall of the vortex tube. Case 1 corresponded to vortex length l = 0.32m, generator diameter d_0 = 0.1 m, and angular velocity of rotation of the blades of rpm. Case 2 was observed for l = 0.64 m, d_0 = 0.05 m, Ω = 5900 rpm. In Case 1 a gap was set up at the top of the chamber, and in Case 2 the chamber was tightly covered. Figures 1 and 2 show the results of velocity measurements in Case 1 at height z = 0.1 m, and in Case 2 at z = 0.14 m (Fig. 1 shows the tangential velocity (curves 1 and 2) and the circulation (curves 3 and 4) in the vortex tubes; 1 and 3 indicate a vortex tube of one-cell structure; 2 and 4 show a two-cell structure; in Fig. 2 curve 1 is a one-cell vortex, and curve 2 refers to a two-cell vortex). The velocity components v_φ, v_z were measured. On the graphs these quantities are referenced to the vortex core velocity v_0. The distances from the vortex axis in Figs. 1 and 2 were reckoned in dimensionless units, obtained by dividing by the characteristic dimension r_0, the core radius.

In a one-cell vortex there are ascending flows along the axis, and in the two-cell vortex the axial flow is descending. In the one-cell vortex the particles approach the

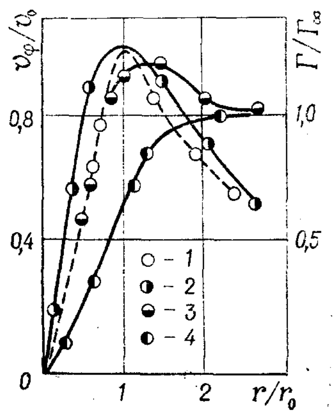

Fig. 1

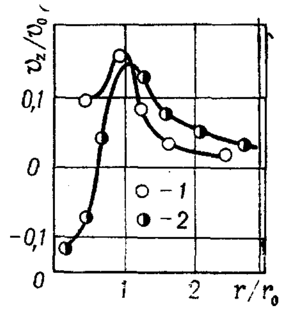

Fig. 2

TABLE 1

Parameter / Vortex type	One-cell (measurements of the authors)	One-cell [10]	Two-cell (measurements of the authors)	Two-cell [10]
Γ_∞, $10^4 \frac{cm^2}{sec}$	7,9	11,0	1,5	16,7
v_0, $10^2 \frac{cm}{sec}$	24,4	19,8	10,8	20,1
r_0, cm	3,8	7,6	1,7	8,4
l, cm	32	183	64	183
Δp, Pa	1300	3800	350	500
Q, $10^4 \frac{cm^2}{sec}$	15,9	44,0	2,3	4,0
Re_r, 10^4	96	18,8	2,5	1,6
ε	0,53	0,53	0,90	0,029
s	0,24	0,33	0,59	0,65

center, and in the two-cell vortex the particles move from the center towards the edge within the core. In the potential part of the vortex the particles also approach the edge of the core. The axial flow is formed as a result of flow towards the edge of the core.

Calculations of the circulation $\Gamma = 2\pi r v_\varphi$ at different distances from the vortex axis show a different dependence for the two cases. Figure 1 shows values of the circulation referenced to the circulation Γ_∞ measured far from the vortex axis. A noticeable increase of the circulation is observed in the two-cell vortex. It occurs in the region of trnasition from the solid rotation axis to the potential flow.

Reference [10] has proposed the dimensionless parameter $\varepsilon = Q/R\Gamma_\infty$ to evaluate the structure of flows in the vortex tube, and here the flow rate Q varies with height, since the air in the upper part of the vortex generator, drawn out from the vortex, moves away through the exhaust tube. The amount of air drawn off from the vortex was determined from the vertical velocity v_z at the height where the radial flows do not vary with distance from the bottom and are practically zero, i.e., at entrance depth h. In our vortex model the value of Q was determined from data of measurements of v_z at a height equal to the depth of flow entrance into the vortex.

Table 1 shows the results of calculating the parameters describing the structure of the two types of vortex.

According to what was said in [10] for the transition from a one-cell vortex to a two-cell type one must achieve a considerable reduction of the parameter ε. It can be seen from Table 1 that this condition is not observed. In the two-cell vortices excited in our measurements, the parameter ε had a value larger than in the one-cell vortices. It was shown in [7] that transition from a one-cell to a two-cell vortex occurs at a critical value of the vortex ratio calculated from the formula $s = 9.04\, a^{-0.05} Re_r$, where $a = h/R$ is the depth of flow entry into the vortex. The one-cell vortex was observed experimentally for $s < 0.45$. Our measurements, as can be seen from Table 1, confirm this conclusion.

LITERATURE CITED

1. M. A. Lavrent'ev and B. V. Shabat, Hydrodynamic Problems and Their Mathematical Models [in Russian], Nauka, Moscow (1977).
2. É. P. Volchkov, V. I. Kislykh and I. I. Smul'skii, "Experimental investigation of the aerodynamics of a vortex chamber with end blowing," in: Structure of the Wall Boundary Layer [in Russian], Izd. In-ta Teplofiziki, Novosibirsk (1978).
3. B. A. Lugovtsev, "Turbulent vortex rings," in: Unsteady Problems of Hydrodynamics, No. 38 [in Russian], In-ta Gidrodin. Sib. Otd. Akad.Nauk SSSR, Novosibirsk (1979).
4. M. A. Gol'dshtik, Vortex Flows [in Russian], Nauka, Novosibirsk (1981).
5. V. A. Vladimirov, "Stability of flows of the tornado type," in: Dynamics of Continuous Media, No. 37 [in Russian], Izd. In-ta Gidrodin. Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1978).
6. V. V. Nikulin, "Modeling of tornado-type vortices," in: Mathematical Problems in the Mechanics of Continuous Media, No. 47 [in Russian], Izd. In-ta Gidrodin. Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1980).
7. R. Davies-Jones, "Laboratory simulated tornadoes," Proc. Symp. on Tornadoes, Tech. Univ., Lubbock, Texas (1976), p. 151.
8. J. S. Turner, "The constraints imposed on tornado-like vortices by the top and bottom boundary conditions," J. Fluid Mech., 25, No. 2 (1966).
9. R. A. Granger, "Laboratory simulation of weak-strength tornadoes," J. Fluid Mech., 3, No. 4 (1975).
10. C. A. Wang and C. C. Chang, "Measurement of the velocity field of a simulated tornado-like vortex using a three-dimensional velocity probe," J. Atm. Sci., 29, No. 1 (1972).
11. N. B. Ward, "Exploration of certain features of tornado dynamics using a laboratory model," J. Atm. Sci., 29, No. 6 (1972).

EXCITATION OF TOLLMIEN–SCHLICHTING WAVES IN THE BOUNDARY LAYER BY THE VIBRATING SURFACE OF AN INFINITE SPAN DELTA WING

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The problem of instability wave origination (Tollmien-Schlichting waves) is discussed extensively at this time in connection with the solution of the problem of predicting the laminar-to-turbulent boundary layer transition point [1, 2]. The problem of exciting Tollmien-Schlichting waves is considered in [3] in the case of a two-dimensional boundary layer on a vibrating surface. This paper is devoted to the solution of the problem [3] in the case of spatial perturbations in the boundary layer in the vibrating surface of an infinite span delta wing.

1. FORMULATION OF THE PROBLEM

Let us consider the flow in the boundary layer on an infinite span delta wing. We select as coordinate system: x is the distance from the leading edge along the streamlined surface, y is the distance along its normal, and the Oz axis is along the wing leading edge. We write the Navier–Stokes equations in dimensionless form by using a certain length scale l, and the free stream velocity U_0. We measure the time in the units l/U_0, the pressure is referred to $\rho_0 U_0^2$ (ρ_0 is the density in the free stream). The temperature and the viscosity coefficient are also measured in units of the corresponding quantities in the free stream. As in [4], we assume that the fundamental flow is weakly inhomogeneous in the absence of perturbations. The following dependence on the coordinates is assumed for the velocity components (U, V, W) and the pressure and temperature (p, T):

$$U = U(x_1, y), \quad V = \varepsilon V_*(x_1, y), \quad W = W(x_1, y),$$
$$p = p(x_1), \quad T = T(x_1, y), \quad x_1 = \varepsilon x, \quad \varepsilon \ll 1. \tag{1.1}$$

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